

The Unlucky Digit

Your unlucky digit is three (3). The school you will enter next year has purchased 300 new lockers that have been pre-numbered with 101 through 400. They will assign you a new locker at random. What are your chances of avoiding the digit three (3) in your locker number?

Bonus: Your unlucky digit is instead 4. You will pick any positive integer, from one to infinity, at random. In what interval do you have less than a 0.5 probability to avoid any appearance of the digit 4 in the decimal representation? [Hint: logarithms might be helpful.]

Source: variation on an idea in *Keys to Infinity* [Pickover]

Solutions

Of the 300 locker numbers, there are 138 that contain at least one occurrence of the digit three. The probability of avoiding one of those is $162/300$. The odds are slightly in your favor.

Bonus: The probability of avoiding a 4 in any one decimal place is $9/10$. To avoid it with 0.5 probability in any of n places:

$$0.9^n = 0.5 \text{ or}$$

$$n = \log(0.5) / \log(0.9)$$

$$n = 6.57881348$$

$$10^n = 3791521.115$$

So in the interval $[3,791,522, \text{inf})$ you have less than half a chance to avoid your unlucky 4. It's just too many random digits that must all avoid being four.