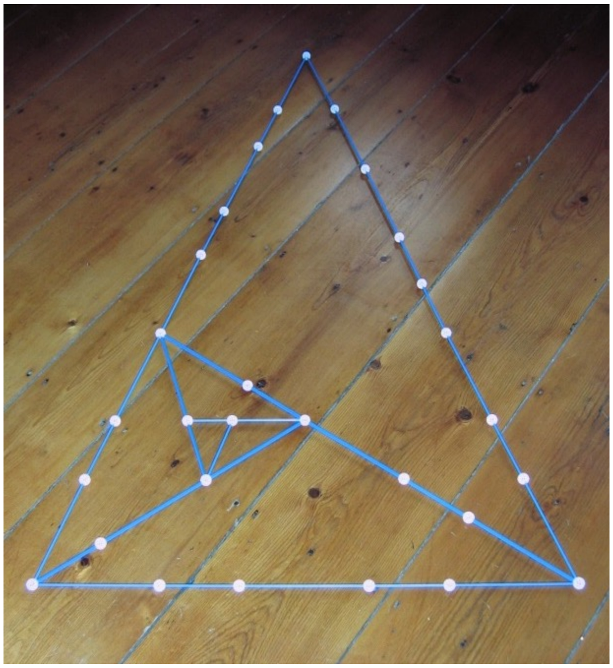
Zome Geometry

Settling into a Flat

“Uncle Bob”

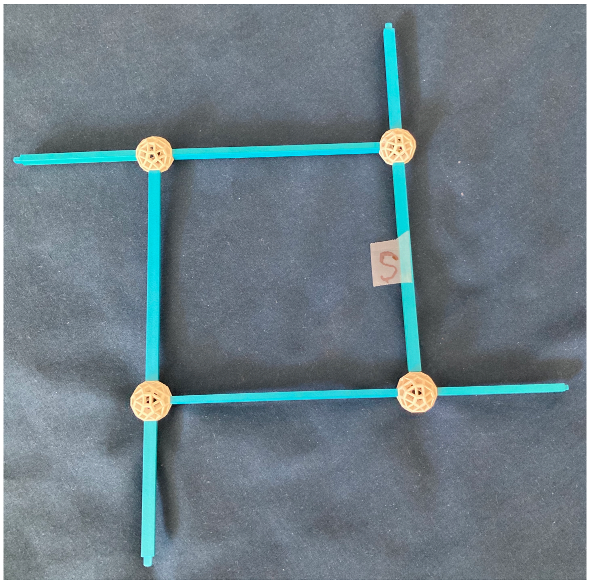
We now settle into the plane with the objective of supporting the two-dimensional models in our **Zome Gallery**. In discussing 3-D models we’ve ingested many of the terms and properties of the flat shapes. Corners are still vertices, faces are polygons, and edges are usually called sides. Angles are acute, right, or obtuse, and a straight angle is the sum of two right angles. We confess that in six lessons we haven’t proven a thing, and that’s due to another idea, that of playing before proving. We intend to make amends here and actually make a few convincing arguments, but first let’s have more examples of play. We leave it for the reader to research Penrose tiles.

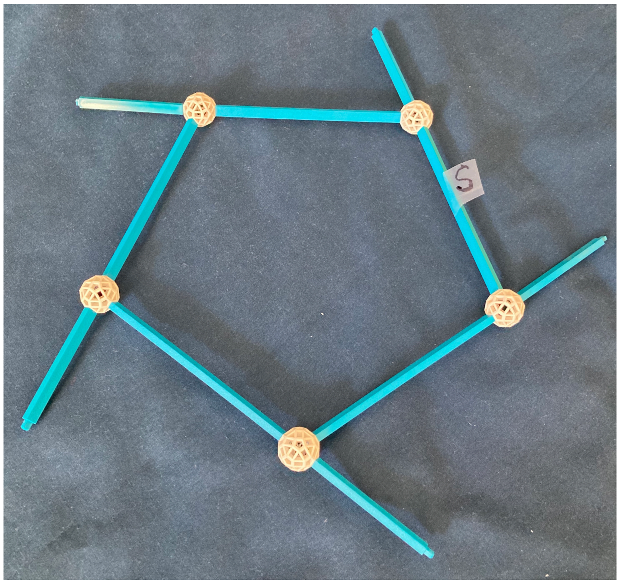
Return to **A Feast of Zomes** and scroll down to view a gallery showing examples of play.



A swirl of Golden Triangles

We assumed that you knew that a polygon known as a square was composed of four equal sides and four interior angles each measuring 90 degrees. We dropped in your lap the fact that a regular pentagon has interior angles of 108 degrees, and an intuitive proof follows.



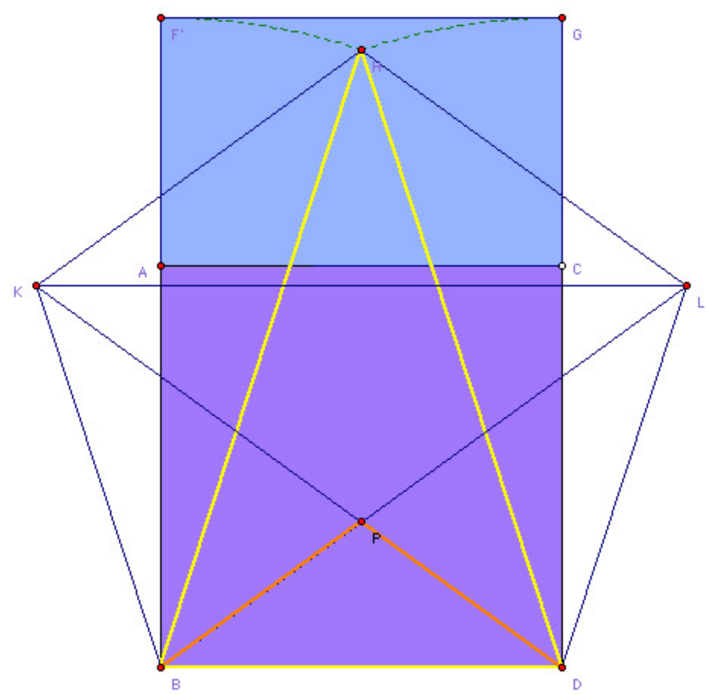


Let's take a walk around the block starting at the point "S." If the block is square we will make four 90-degree turns for a total of 360 degrees. No wonder we end up facing the same direction as at the start. It's like going once around a circle. In both polygons we see an interior and an exterior angle at each vertex. To make the round trip, at each corner we must diverge from going straight by an amount equal to the exterior angle. A pentagonal "round" trip makes five turns which also total 360 degrees. So the exterior angles each measure one-fifth of 360 or 72 degrees. Knowing that the combined pair of an interior and an exterior make a straight angle of 180 degrees, we calculate that each interior angle is 180 less 72, or 108 degrees.

Mining the Gold

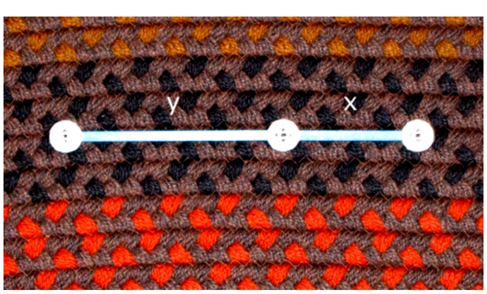
The regular pentagon is innately connected with a quantity commonly known as the Golden Ratio. In the figure at right you can see how a rectangle, a triangle, and a regular pentagon fit together, and how, given one, the other two can be constructed. But what makes their proportions Golden? What is this ratio we call Golden? The answer is even simpler than all you've seen thus far.

Construct Golden Triangle Inside Golden Rectangle and Expand to Pentagon



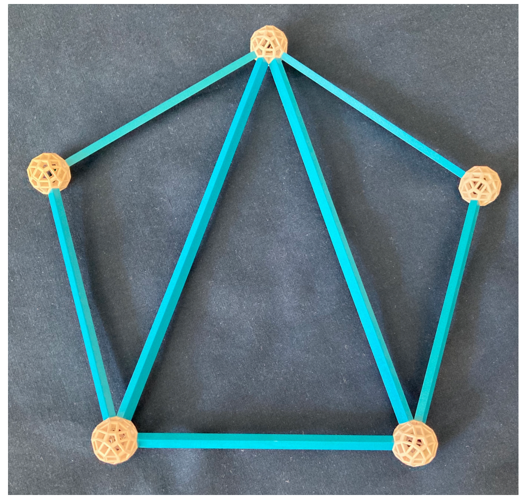
How were pts. K and L found?

Zometool struts were manufactured with the Golden Ratio in mind. For an understanding of ratio we turn to the real world and investments. If you put $1000 into a project that promises a 50% growth each year, first off, be skeptical … but go along here for the sake of math. In one year your thou will be worth $1500, and after a second year you will have 1500 plus half again, or $2250. We can figure it by multiplying the current balance by the growth factor 1.5 which amounts to 3 parts returned for every 2 parts invested, and that's pretty good! The ratio is 3:2 (said, three to two).



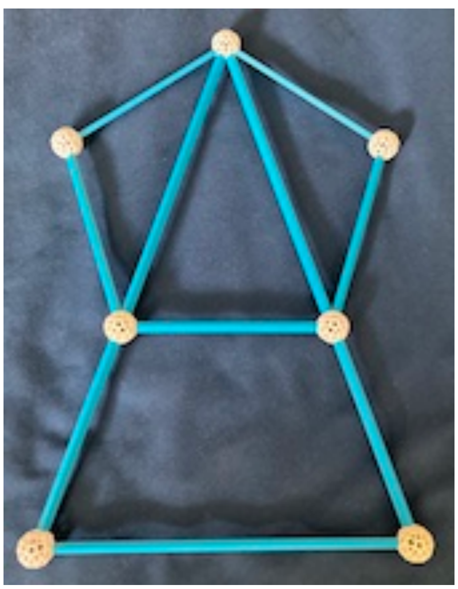
In the photo at left, say the length of the shortest segment represents your initial investment. In one year it grows to the longer portion. In a second year it grows to the sum of both portions. It takes a particular growth factor to accomplish this, and it is approximately 1.62. The ratio is about 1.62 to one. Yes, a Golden investment would be earning about 62% per year.

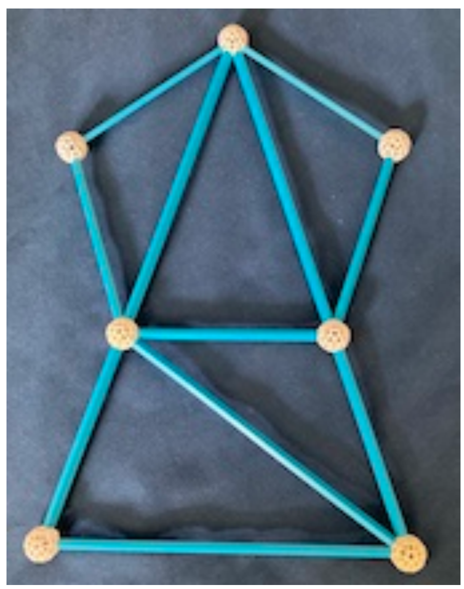
Feet back on the ground, or plane, a Golden Triangle is an isosceles triangle. It has two equal sides. In a GT the ratio of those to the third side is Golden. A Golden Rectangle has its length and width in this ratio. Pictured at right is a Golden Triangle made from a side and two diagonals of a pentagon.



Those two flattish triangles either side of the tall one are Golden as well with the ratio reversed, but here we choose to ignore them.

We want to make a convincing argument that the growth from the short side at the base of the triangle to the longer side, is equal to the growth from the longer side to the sum of both sides. We employ two pics.

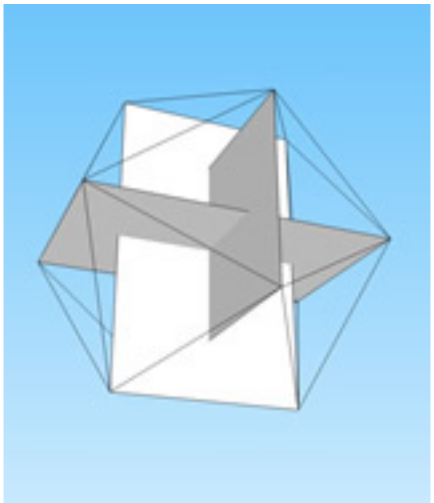




In the first figure we made a larger triangle by extending outside the pentagon by the length of the shorter side. The larger triangle is similar to the one inside the pentagon because they share angle measures. The extended triangle will have its sides in that same ratio, but can we show that the very bottom segment is the same as the pentagon's diagonal? In the second pic a segment has been added to help you see this fact.

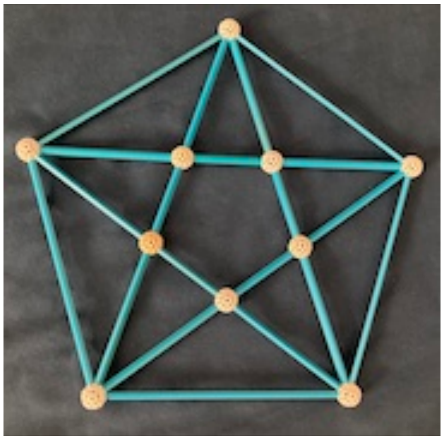
The Golden Rectangle is defined as one with its length and width in this ratio. It is a curious fact that three of these rectangles are contained in one of our Platonic solids — the icosahedron.

In the next and final lesson, we will defend the more orderly and logically tight system on which formal geometry is based, and upon which we have largely not leaned. We will present a favorite proof in two dimensions that Uncle Bob calls "Order from Chaos." It also employs the properties of similar triangles, so the reader may want to study up — or not.



Tasks (Solutions on page 5)

1. How many different angle measures can you name in the figure at right. You might start with the small pentagon in the center and its interior and exterior angles.



2. Use the method that we applied (to find the pentagon's interior angle measure) to calculate those measures in a regular hexagon and a regular decagon (10 equal sides and angles).

3. Number fans might enjoy these two Golden properties.
a. In a Golden investment one dollar would grow by a factor of about 1.62 and yield $1.62. After a second year of Golden growth, what would the investment be worth.

b. Say we are in a period of deflation. Our $1.62 from last year has shrunk to $1.00. If that dollar shrinks by the same factor next year, what will the balance be?

Solutions

1. The small pentagon has interior angles of 108 degrees, and exterior angles of 72. The star points are golden triangles with two angles of 72 and a third of 36 degrees to make a sum of 180. In between the star points are flat isosceles triangles with an obtuse 108-degree angle (can you see why?) and the balance of 72 split equally between the two other angles. Other angles are combinations of these measures.

2. A regular hexagon has 6 interior angles each measuring 120 degrees. Multiply 6 by 180, subtract the 360 exterior total, and divide the rest by 6. A decagon has interior angles of 144 degrees each.

3 a. Recall that golden growth results in the sum of a smaller and a larger part, so as $1 grew to 1.62, 1.62 would grow to about $2.62. Check that by multiplying 1.62 by itself.

3 b. Working in reverse, a drop from $1.62 to a buck results from dividing by the factor 1.62, and 1/1.62 is approximately 0.62. We could get the result by subtracting one from 1.62. Only the Golden Ratio relates addition and multiplication in this way.