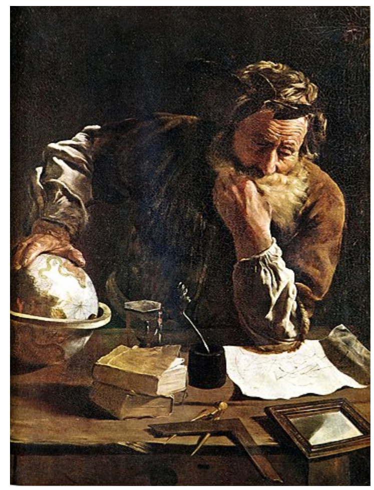
Zome Geometry

Archimedes Steps Up

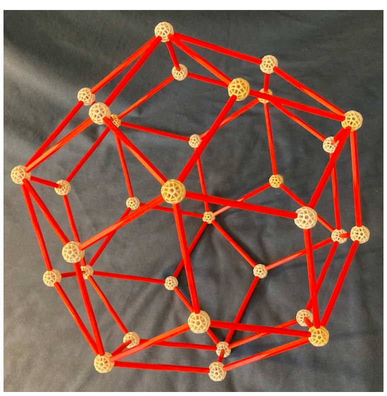
"Uncle Bob"

This is the fifth article in a series serving as a companion to Uncle Bob's Zome Gallery. To this point we have used the properties of angle deficit and the Euler [said "oiler"] number to shorten the task of counting the faces, vertices, and edges of some well-behaved three-dimensional figures. We have looked at the Platonic solids [see Zome Perfect], a set of five deemed regular in all respects, i.e., having identical faces with equal sides and angles, and vertices of an identical makeup. In this article we introduce the Archimedean solids, and in doing so, add a little variety to our 3-D collection.

Archimedes, the famed mathematician and physicist, was no amateur in the field of solid geometry. Among his many accomplishments were the calculation of the volume of a sphere and how its measure compares to circles, cylinders, and cones of similar dimensions. Evidence suggests that he looked at the Platonic solids and asked a question dear to all mathematicians: "What if?" In this case, what if we allow most, but not all, of the Platonic properties? Is there a second tier of nearly ideal solids, and which properties will we stipulate and which can be relaxed?

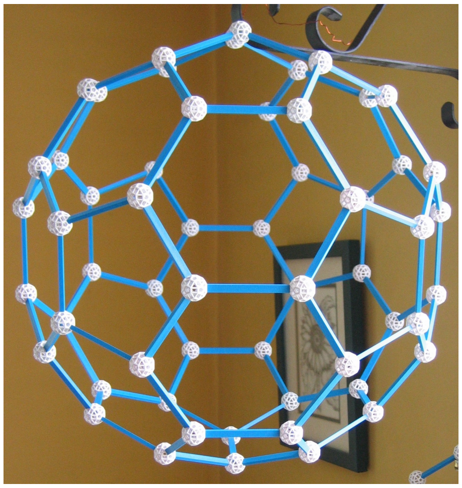


Archimedes by Dominico Fetti - 1620
public domain in the U.S

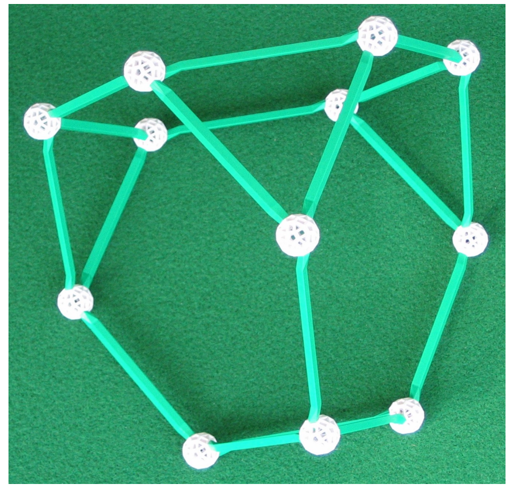


In our fourth article [see The Teaser] we analyzed a solid, the rhombic triacontahedron at left (my spell checker doesn't like it still). It has identical edges and faces, but does not qualify as Platonic. It has two different face angles, one acute and one obtuse, and two different vertex compositions, joining five edges or three. Neither does it qualify as Archimedean.

Archimedes searched for and found a set of 13 convex solids that were composed of faces of regular polygons meeting in identical ways at each vertex. What property was relaxed? Answer: the faces needn't be all the same polygon, but those polygons need to have an identical share at each vertex, and in order for the faces to join along whole edges, those edges have to be the same length. We have worked with two such Archimedean solids already.



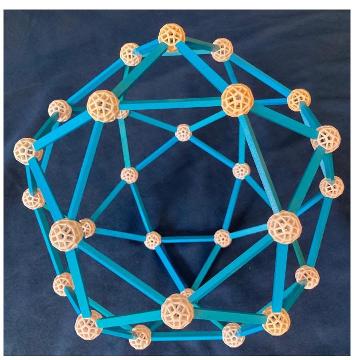
[5, 6, 6] See What's to be Gained …



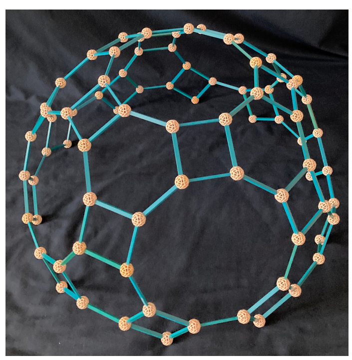
[3, 6, 6] See Parts is Parts

As strange as the shape above on the right appears, it meets all criteria: it is convex (has no dents), it is composed of equilateral triangles and regular hexagons, and every vertex is configured in the same way.

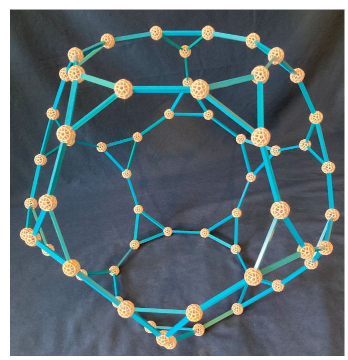
It is the vertex configuration that allows us to name these solids very concisely. Above you see the [5, 6, 6] and the [3, 6, 6]. By convention the numbers begin by indicating the least polygon and then follow a rotation around a vertex naming the others. As mentioned, Archimedes found thirteen such shapes. Here are some others:



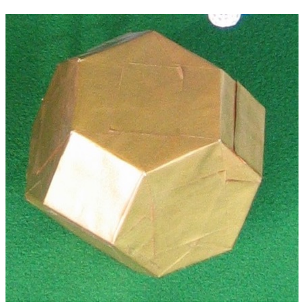
[3, 5, 3, 5]



[4, 6, 10]
incomplete



[3, 10, 10]



[4, 6, 6]

Truncation

Whisper this term, please. It could cause an elephant stampede if we're not careful. It is possible that Archimedes discovered some of this set by visualizing the truncation of each vertex of a simpler solid. In this context it means the shaving off of vertices to create other regular polygons in place of the original vertices and faces. It is in fact the method by which some of these Zome models were created from others, but it may be simpler to illustrate the method with an actual solid and an actual — shh — truncation. See tasks, below.

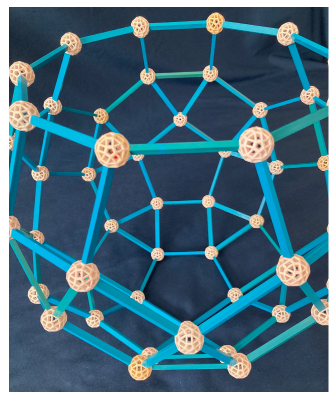
Archimedes is known to have written a book on this family of solids, but the book is lost and may never be found. He may have had even more insights into this topic, but that knowledge is likely lost forever. Two libraries were destroyed by fire in ancient times. Brilliant minds in our own age are making astoundingly beneficial discoveries, but this new knowledge is not guaranteed to hang around in perpetuity — a thought I try to impress on my students. Knowledge must be preserved, utilized, and shared to fulfill such a guarantee.

The next and penultimate article in our series will look at relations among the solids.

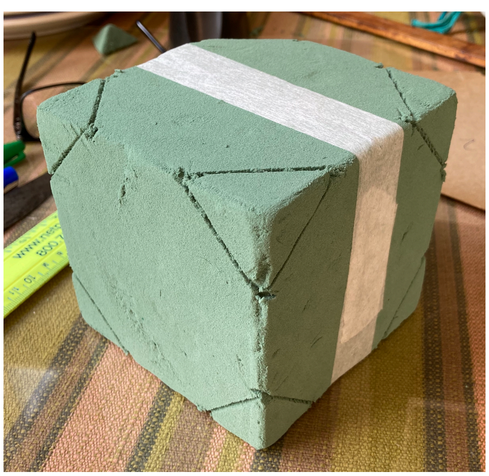
Three Tasks

(answers on page 4]

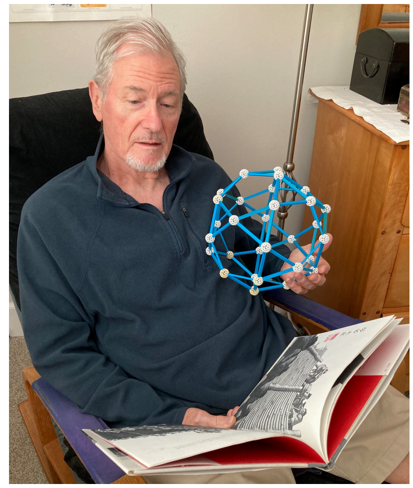
1. Name this partially completed Archimedean solid at right by bracketing four numbers in the correct sequence.



2. We are about to truncate vertices of the cube to form an Archimedean solid. Can you say what new regular polygon will be formed by shaving the corners? Can you say what new regular polygon will remain on each face? Can you name this Archimedean solid?



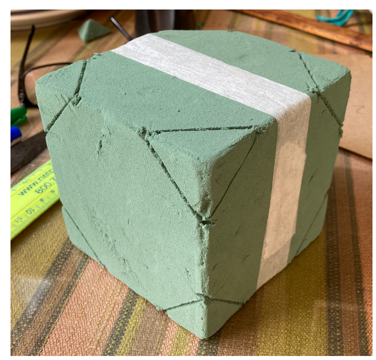
3. The [3, 5, 3, 5] is one of my favorites — so round and cuddly. What is the angle deficit at each vertex? How many vertices are there?



Solutions

1. [3, 4, 5, 4]

2. The truncations will leave equilateral triangles at each vertex and regular octagons at each face. We shall label this solid [3, 8, 8].



3. At each vertex the [3, 5, 3, 5] has 2x60+2x108 or 336 degrees — a deficit of 24. The solid should have 30 vertices to make the 720-degree total deficit.