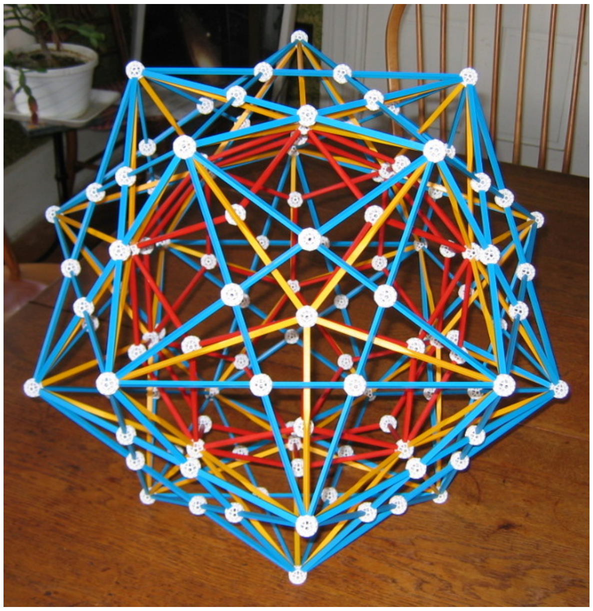
Zome Geometry
The Teaser
"Uncle Bob"

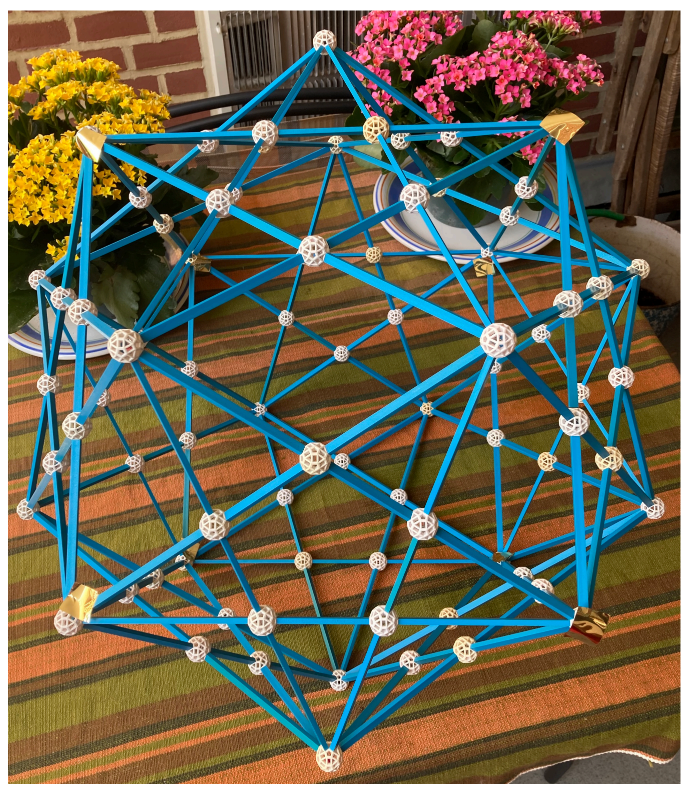


Earlier in this series, when we were learning some shortcuts for counting the faces, vertices, and edges of polyhedra, in an article titled "Parts is Parts," we used the figure at the left as a teaser to suggest — threaten? — that we count its parts. The Teaser's real name is the Compound of Five Cubes, and it is built on other polyhedra serving as a scaffold. Before we unpack it for you, let's make some observations from **the slo-mo movie of this shape**.

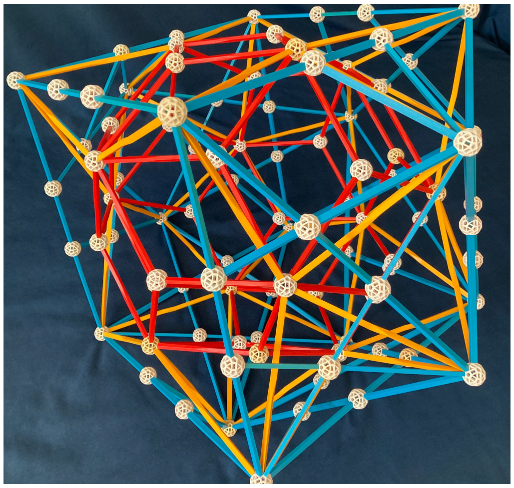
The things to look for are:

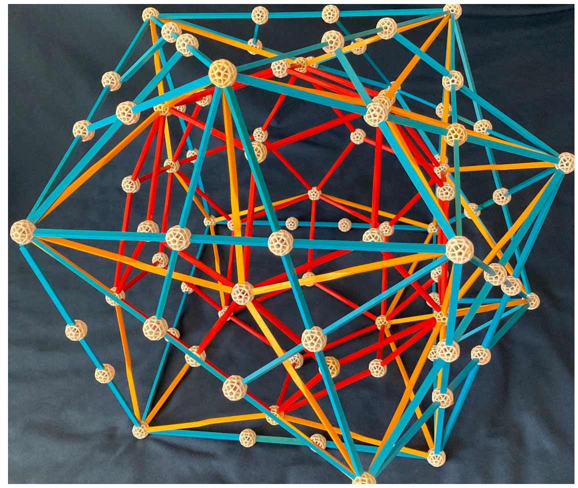
- Pentagrams: five-pointed stars in blue
- Concave yellow stars reaching to the points of the pentagrams
- The red polyhedron at the center
- Perhaps most difficult to spot are the five cubes, large and blue, and tumbling in five orientations.

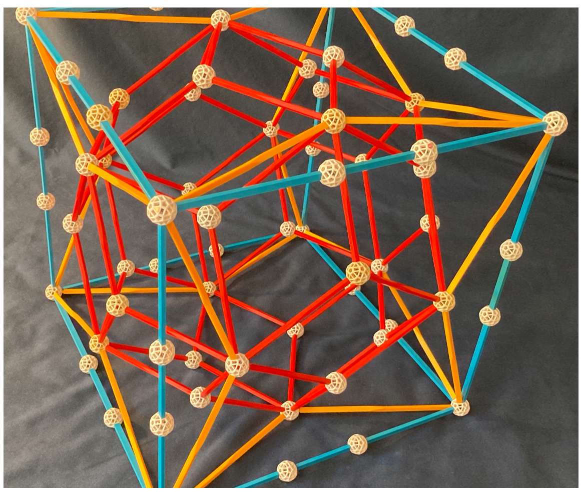
The cube edges are all segmented because, in the jumble of orientations, they all intersect other edges. OK. The five distinct cubes are difficult to distinguish, and they are almost as difficult when presented without the scaffolding. I placed some gold foil tape on the corners of one of the cubes in the picture at right …

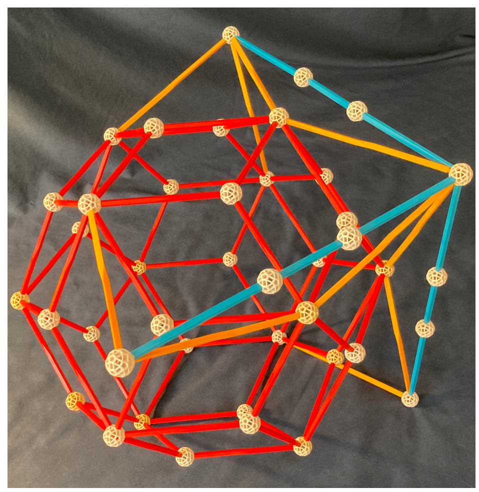


… and now we strip our Teaser model bare …







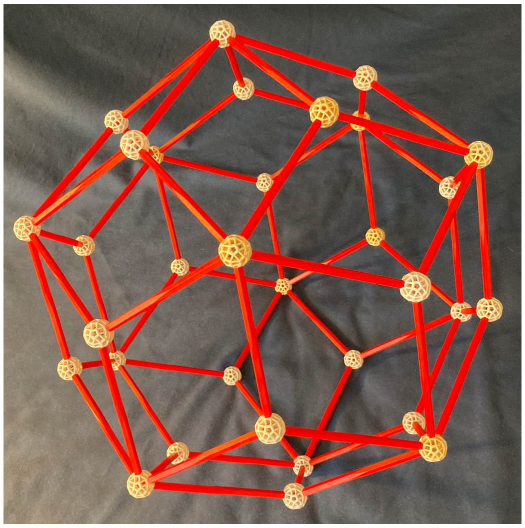


A Return to the Parts Department

In order to do a little review of counting methods, let's count up the struts, balls, and faces of the red model found at the Teaser's core — pictured on page 3. Its name is rhombic triacontahedron. Each face is an identical rhombus. Thinking about angle deficits [see What's to be Gained from Deficits?], we immediately encounter two problems. Each rhombus has two different face angles, and the model has two types of vertices — one type where three struts meet, and another where five do. How will we utilize angle deficit and the Euler number [see Zome Perfect] to count the parts?

We could be slick and use Pythagoras and trigonometry to nail down the exact measures of the two face angles, one acute and one obtuse, BUT we won't. We'll fly by the seat of our pants.

I found that the acute angle was a very tiny bit larger than the 60 degrees of an equal-sided triangle. I estimated 62 degrees. The obtuse angle, in any parallelogram or rhombus, will be the supplement, about 118 in this case. Three angles of 118 meet at some corners. The angle deficits at those are approximately 360 – (3 x 118) or 6 degrees. Those corners are almost flat and much flatter than the 5-way corners, each with a deficit of 360 – (5 x 62) or 50 degrees each.



This red polyhedra is convex and has no holes or serious dents, so its total angular deficit should be 720 degrees, but how many of each vertex does it have? A little algebra, which we shall skip, will show that for these to be whole numbers, the count of five-way corners must be a multiple of three. Studying the image at left, I see more than six of these corners, and reason that if they pair up as polar opposites there would be 12 and not nine.

See it in motion.

By the way, the compound of five cubes (pictured at the top), together with its red rhombic core and yellow scaffolding, requires 112 balls and 300 struts — but my counts may be off a little. Better check me.

Tasks

Solutions on page 4

All questions pertain to the red core.

1. Twelve corners (vertices) each have an angle deficit of 50 degrees. Their total deficit comes to 600 degrees. How many corners of deficit 6 degrees are there? How many corners are in the entire model?

2. Twelve corners have five edges each, and 20 corners have three. Multiplying, we get 60 + 60 or 120 edges, BUT this over counts because each edge requires two corners. How many edges are there in the model?

3. Faces. We can now use the Euler number (2) to calculate the number of faces.

faces + vertices – edges = 2

How many faces hath the model?

Solutions

1. 720 – 600 leaves a deficit of 120 for the remaining corners which each have a 6 degree deficit. There must be 20 of those and 32 vertices in all.

2. 120 double counts the edges which are 60 in all.

3. faces + 32 vertices – 60 edges = 2. The faces must number 30 in all, and thus the cataloguers have named this solid a rhombic triacontahedron — and my spelling checker doesn't like it. Tough, I say.

In our next installment Archimedes will broaden and enrich our topic.