# Zome Geometry

Order from Chaos

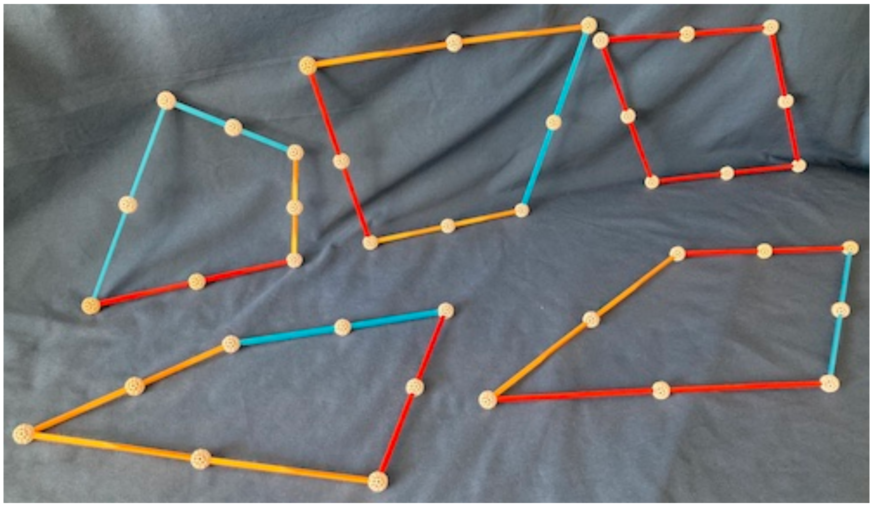
"Uncle Bob"

Aunt Claire and I hope that you are enjoying your romp through a bit of geometry. We relied heavily on the attractive and precise Zometool models and an excellent activity book written by Hart and Picciotto. We have been experimenting with the idea that starting with 3-D geometry is no more difficult and perhaps even more palatable than dealing with all the flat stuff. Perhaps in your schooling you got bogged down with proofs in the plane and never got to these beautiful solids and their properties and relationships. In our lesson seven "Settling into a Flat," and in this final one, we introduce plane figures, in part, to support some of the items in our Zome Gallery.

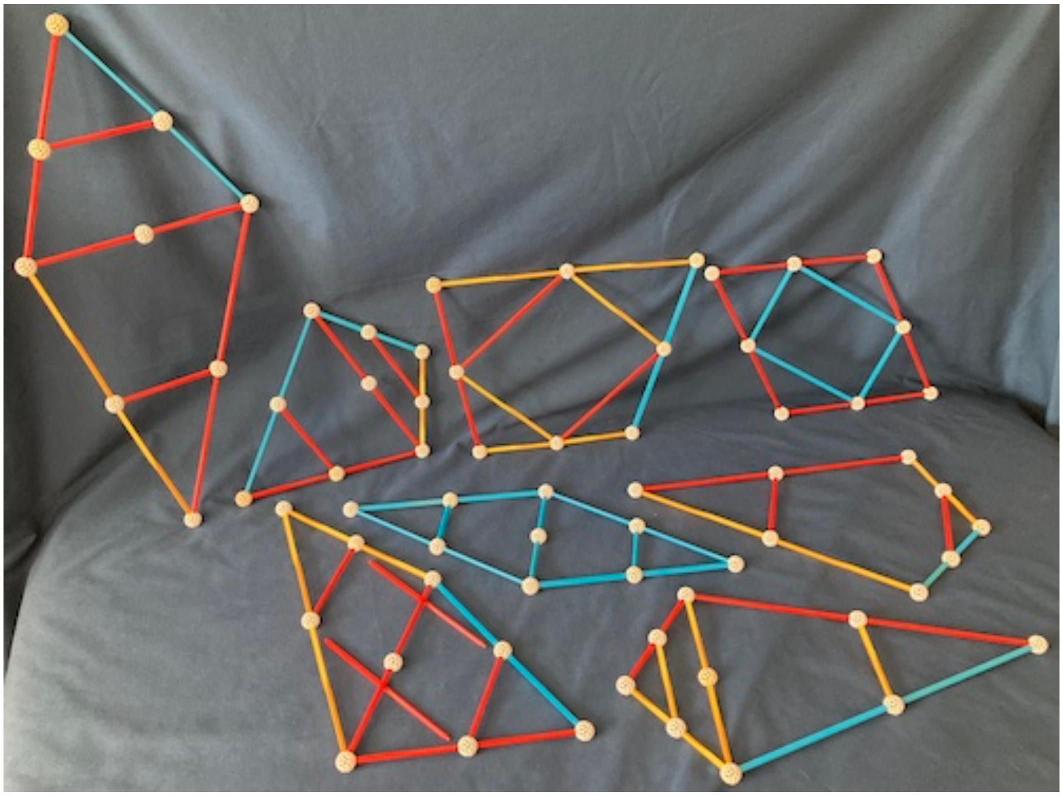
The subject of geometry was installed permanently in schools in the middle ages. It was one of three or four essential branches of study. Its rank in importance is not due to facts like the sum of the measures of three interior angles in a triangle is the same as the sum of two right angles. The importance lies in the disciplined approach to the subject. Nothing seen or exemplified is assumed to be generally true. Measuring is one way to approximate and suggest a fact, but a measurement cannot be taken as an exact indication of truth.

Geometric truth must come from proofs based on an understanding of basic terms and a few sensible assumptions, precisely defined terms, and an orderly sequence of previously proved truths. The discipline employs a logic which can be carried over into a professional career or a technical craft to great advantage.

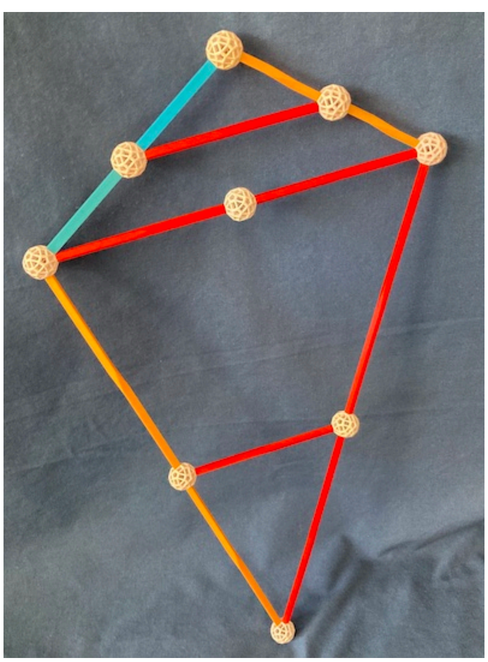
We'll conclude our shapely romp with one more geometric fact which we will try to establish in a disciplined way. I call this fact "Order from Chaos." It states that by joining the midpoints of the adjacent sides of any four-sided plane figure we create a parallelogram — possibly a more "orderly" quad with opposite sides parallel? Here we see a motley collection.



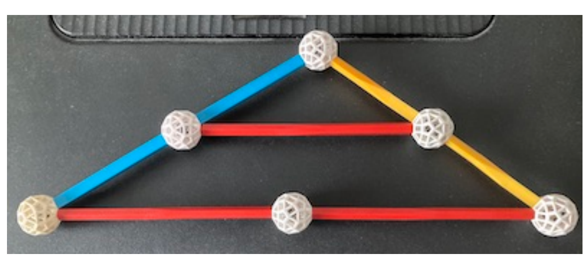
As varied as this set is, it cannot represent every quad. For instance, there is no rectangle in the bunch. Our theorem says that if we join adjacent sides at their midpoints we will get a parallelogram. Let's see.



In the set above we have joined the midpoints where Zometool allowed. We call the connections midlines. A set of four midlines appears to make a parallelogram, but looks don't prove — any more than measuring does. Note also that in some of the figures we have connected not only midpoints, but opposite corners as well. They divide the quads into two triangles. These segments appear to be parallel to the midlines, and if they are then the midlines must be parallel to one another. The proof about quads reduces to establishing that the midline joining two sides of any triangle is parallel to the third side.



This fact is established by showing that the two nested triangles in the figure below are similar because they share an angle included by two sides that are in a two-to-one ratio. If the triangles are similar then the other pairs of angles match up, and if they do, then the third sides are parallel. This is true in general because the logical chain used did not depend on any particular triangle.



In joining the midpoints of the quads we have created parallelograms inside — made order from chaos, if you will. The implication statements — “if this is true, then this is true” — allow us to prove more facts from previously established ones, and build a geometry out of nearly thin air, into a geometry that keeps our town squares square, our garden rows parallel, and our skyscrapers vertical.