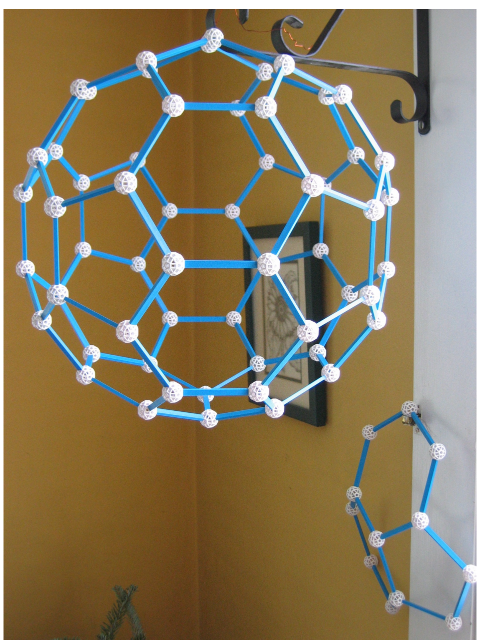
Zome Geometry: What's to be Gained from Deficits?

Uncle Bob

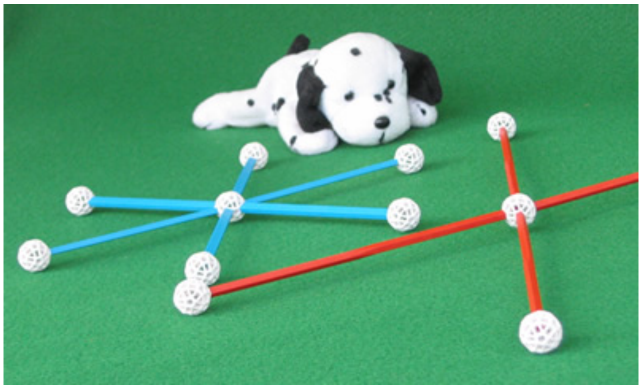
If I were teaching a class of Geometry today, my first lessons would be — contrary to most texts and curricula — in three dimensions and not two. Of course, in many public schools today, this act alone would get me a pink slip, or a performance review at the least. Ah, Retirement!

I don't regard starting with solid figures as a track down from more complex to simple flat figures. The world we live in and deal with is mostly three-dimensional — more cube-like than square-like. If we order some squares from Amazon, perhaps party napkins or coasters, they will come in a box. We must deal with the box before we touch the squares. I view my track from solids to flats as going in a sensible way from reality (throw me the sphere, Charlie) to abstraction (the sum of the interior angles of a triangle is two right angles). Get my point?



After introducing a few terms like point, line, vertex, face, and edge, I might have my first unit of instruction, in several lessons, tackle angle deficit. It is the property essential to the formation of solid figures such as the polyhedron (5, 6, 6) in the picture. Polyhedrons are solids that have flat faces, such as the pentagons and hexagons you see. It's nice, but my track would start with the easiest polyhedron to understand — the cube, or (4, 4, 4), if you'll allow.

What are these numbers about? Well, a polyhedron's faces are surrounded by straight edges (modeled by Zome struts), and those edges meet at a corner, formally know as a vertex. The nice feature of these solids is that they are sort of regular. The cubes faces are all 4-sided squares and three squares meet at every corner: (4, 4, 4). The hanging figure at the left has a regular pentagon and two regular hexagons meeting at each vertex: (5, 6, 6). The property that allows these faces to wrap around and make closed solid figures is termed angle deficit. Click **here** to see the Archimedean solid 5, 6, 6 in motion.



Sparky, our fuzzy Dalmatian, will illustrate. He's lying prone and so are the blue and red figures. The central vertex in each (what the Zome people call a ball) is surrounded by angles totaling 360 degrees — four nineties or six sixties. There is no curl made by the faces at the vertex and no chance to make a solid "throw me the ball-type" object. These are great patterns if you want to tile a floor or a wall. Contrast this with the hanging object above. Each vertex is surrounded by the angles of one pentagon and two hexagons and those three angles total less than 360. A deficit.

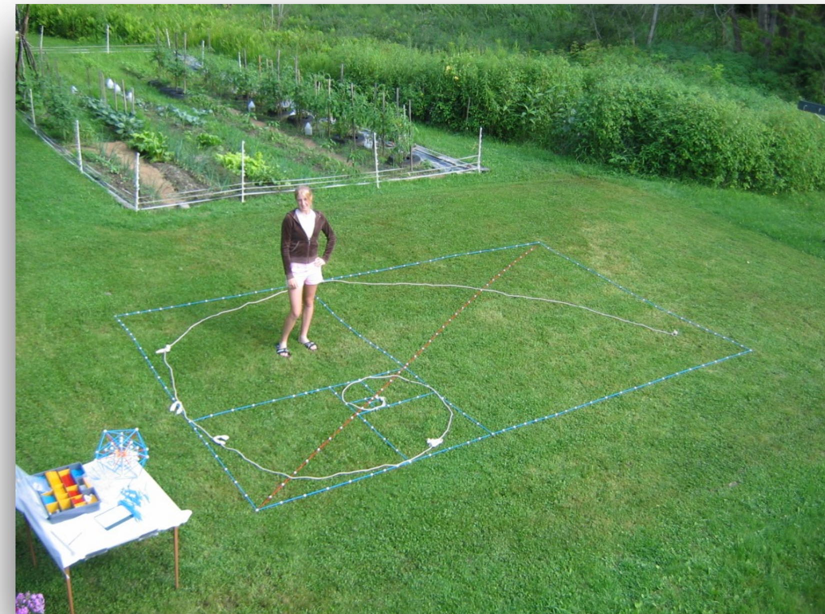
We will learn to calculate many of these deficits, but we can solve for one immediately. A square face has angles measuring 90 degrees, and three of those meet at each cube vertex. Three nineties total 270 degrees and that is 90 degrees shy of 360. So 90 is the deficit at each corner. An amazing fact now: for every well-behaved polyhedron, and these two are nice, the sum of the deficits over every vertex is a constant number of degrees.

Your posers (Answers on page 3)

1. How many corners (vertices - VER-teh-sees) does a cube possess? What is the total deficit overall? This is the total deficit for all "nice" solids with flat faces.

2. The (5, 6, 6) solid pictured above has 60 vertices. Using the total overall deficit from question #1, calculate the angle deficit at each corner of the (5, 6, 6). If a regular hexagon has an angle of 120 degrees at each corner, called a face angle, calculate the number of degrees in each face angle of a regular pentagon (a multi-step problem).

Conclusion.
The teaching of geometry is not the primary issue here, rather it is the learning of it. The solid figures are more tangible and more interesting than the flats — and I think Sparky is all kinds of cute. You'll have noticed that we did have to dip into the world of flat to employ concepts such as square, pentagon, and face angle, but this points to another advantage of tracking from reality to abstraction. By the time we get to straight angles, interior angles of parallel lines, Pythagoras, rhombi, similar figures, exterior and interior angles, and the like … we will have touched on most of it — be grounded in it, if you will.



Answers to the Posers

1. A cube has 8 corners (vertices). If the deficit at each is 90 degrees, the total deficit is 720 degrees.

2. Since the (5, 6, 6) solid has a total deficit of 720 over 60 vertices, there is a deficit of only 12 degrees at each one. A low-deficit corner is more rounded (less pointy). A deficit of 12 degrees means that the face angles of two hexagons (240) and one pentagon total 348; thus the pentagon face angle contributes 108 degrees.

These and other images can be found at our Feast of Zomes Gallery.