

Orbit Posers

A companion to Journey 3: Boy, Devil, Fractal ~ Parts 1 and 2

Here is a number trick that requires the one-time use of a hacksaw. I will lead you to the exact number of poker chips I have in my pocket. Pick any number — a small number if you like. Add seven to your number, then multiply the sum by ten, and then add 25 to that product. Divide the result by 5. When you have the quotient, deduct 14, and next, I insist that you calculate half of what's left.

Am I having a "senior moment"? I just asked you to take half of an odd number. This trick looks like it has fallen on its face, but recall it began with a hacksaw. Perform the final step which is to deduct your original number. Now the trickster produces exactly 2 and 1/2 poker chips from a pocket. Voila! It's a really effective trick when played for a group. When you play it, you can neglect to mention the hacksaw.

We have just sent a number into "orbit," for example, if a person had started with 4, called the "seed," then the following will have been its orbit:

4, 11, 110, 135, 27, 13, 6.5, 2.5,

and 2.5 acts like a black hole attracting all the orbits.

The posers below give you different sets of rules by which you can generate interesting orbits. The solutions follow on page 2.

Orbit #1

You begin with a two-digit seed like 31. Square each digit and find the sum ($9 + 1 = 10$). If you repeat the same steps on the new number, one type of orbit results. Our example leads to 10, then repeating the steps leads to one (1) and gets stuck there. Try some other seeds. Can you find a second type of orbit? Repeat the steps until you see the ultimate fate, which might be described by "what goes around comes around."

Orbit #2

Begin another orbit with the seed pair 1, 2. Make a fraction equal to one plus the second number divided by the first number. That makes $(1+2)/1$, or 3. The orbit so far is 1, 2, 3. Continue making fractions with the two most recent numbers in the list. The rule is: one plus the most recent number over the second most recent. The fourth entry would be $(1 + 3)/2$. This orbit is another example of "what goes around, comes around."

Orbit Solutions

Orbit #1

There is a second type of orbit in addition to the (31, 10, 1) type. Other two-digit seeds fall into a "whirlpool" of eight numbers and never escape it. Starting with 58 the period-8 cycle is 58, 89, 145, 42, 20, 4, 16, and 37. Then 37 leads to 58 and the cycle begins again. Other numbers can enter at any point in the whirlpool and be trapped. Verify this using seeds of 18 and 55.

Orbit #2

The orbit 1, 2, begins a period-5 cycle: 1, 2, 3, 2, 1, 1, 2 ... Other seeds also result in a period-5 orbit. For example, if we begin with 3 and 5, we get $(5+1)/3$ or 2 for the next element. The full cycle is 3, 5, 2, $(3/5)$ or 0.6, 0.8, and then back to 3 and 5.

More on Orbits and Boy, Devil, Fractal in Part 3.