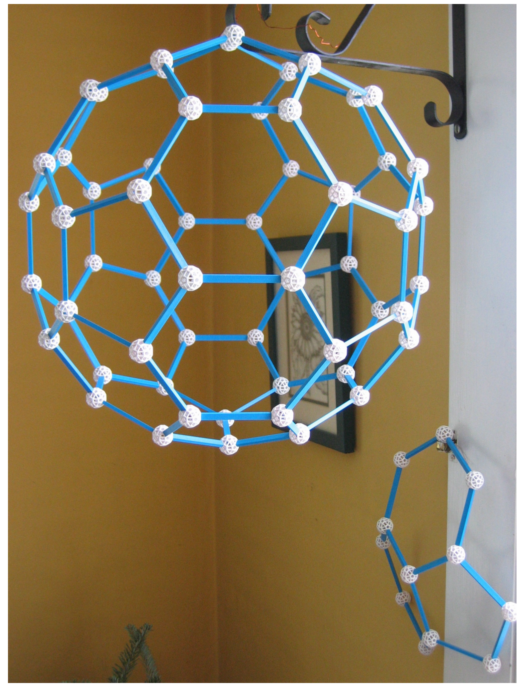
Zome Geometry: Parts is Parts

"Uncle Bob"

You and I, dear reader, work for the Zome Model Construction Company of East Perkasie. We just received an order for 100 units of the (5, 6, 6) model featured in our Zome Gallery and discussed in last month's article "What's to be Gained from Deficits?". Our purchasing agent wants to know how many struts and balls to order — geometrically, these are the edges and vertices of the model.

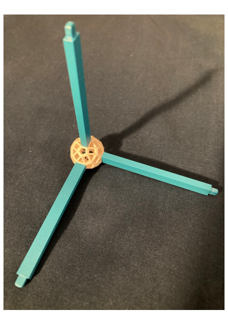


Vertices: We have a jump start on this problem. Again from last month, we know that the face angles of the regular hexagons and pentagons measure 120 and 108 degrees respectively. The three face angles at each vertex total 348 and that is 12 degrees shy of 360. Twelve degrees is the angle deficit per vertex. We also know that solids of this type have a total deficit of 720 degrees over the entire surface. Each corner being identical in composition means that there are 720/12, or 60 such corners, and thems the balls we need to order for each unit.

Faces: Each one of the corners joins two hexagons and one pentagon. Does that mean we have 120 (60 x 2) hexagons and 60 pentagons in the model or have we over-counted? How do you know? How do we correct for this over-counting? (Answers on page 3)

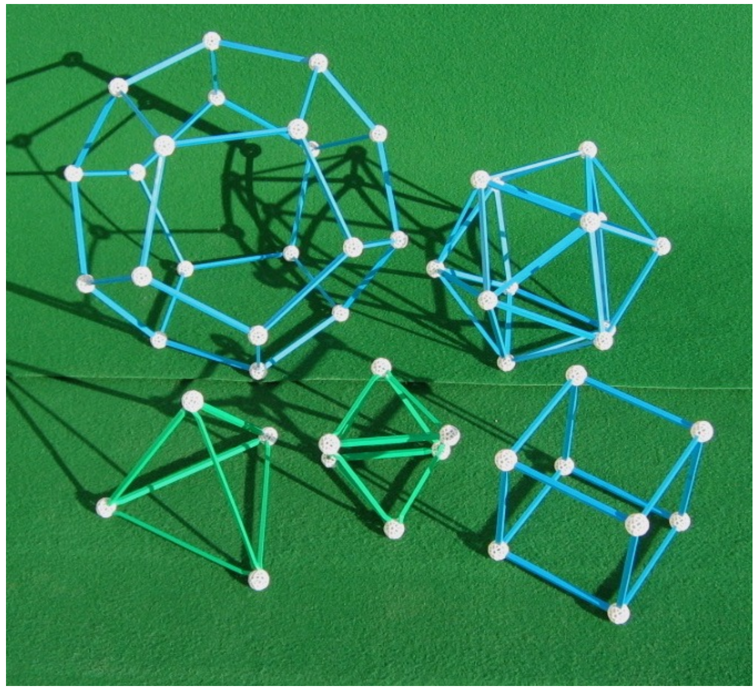
Struts? Twenty hexagons occupy 120 edges, and 12 pentagons occupy 60 edges. Does that mean there are 180 edges in the model? Have we over-counted? How do you know?

Oh ho! The customer has called back and placed an order for 1000 cube models. For this exercise, it will be instructive if you forget anything you ever knew about a cube so that we can invent one and count its parts.



We'll start with the specs that a cube has square faces which meet three to a corner. Refer to the progenitor at the left. The final product again has a total angle deficit of 720 degrees. I leave my shop lackey (that's you) to figure the deficit at each corner, the number of corners (balls) in the cube, and the total number of edges (struts). You get genius points if you avoid looking at or even imagining the entire cube.

Why stop there? At the right you see the frames of a set of five very special solids (details will be forthcoming). For now, know that all the faces in each are the same and have identical face angles. We've covered square and pentagon angles. The triangles have face angles of 60 degrees. Let's label the models, beginning upper left and going clockwise, A, B, C, D, and E. C is the cube, of course.



Put your focus on just one vertex of each model and see if you can complete a table for A, B, C, D, and E listing: 1) angle deficit at each vertex, 2) number of vertices, 3) the number of faces total, and 4) the number of edges total.

Get all that done and we're in business!!

Answers (per unit)

Faces: The (5, 6, 6) solid has 60 vertices, and 2 hexagons at each one, but six vertices are needed for each hexagon. (60 x 2) / 6 = 20 hexagons. By that same reasoning the pentagons number 12. The faces are 32 all told.

Struts: Twenty hexagons occupy 120 edges, and 12 pentagons occupy 60 edges, but each edge does double duty joining two faces. There are 180/2 or 90 edges.

Cubes: The cube corners each have 3 squares or 270 degrees in face angles. The deficit is 360 – 270 or 90 at each corner. It requires 8 corners to make the total 720 deficit. Eight corners each with 3 squares seems to make 24 squares, but squares each require 4 vertices, so we divide 24 by 4 to get 6 square faces (you knew that). There are three edges leaving each vertex, so 3 times 8 is 24, but again we have over counted because each edge requires two vertices. There are 24/2 or 12 edges in the cube.

The table

	Deficit per v	vertices	faces	edges
A	360 – (3 x 108) = 36	20	(3 x 20) / 5 = 12	3 x 20/2 = 30
B	360 – (5 x 60) = 60	12	(5 x 12) / 3 = 20	5 x 12/2 = 30
C	90	8	6	12
D	360 – (4 x 60) = 120	6	(4 x 6) / 3 = 8	4 x 6/2 = 12
E	360 – (3 x 60) = 180	4	(3 x 4) / 3 = 4	3 x 4/2 = 6

Oh. Sales department just called down.
Uh… Uh.…