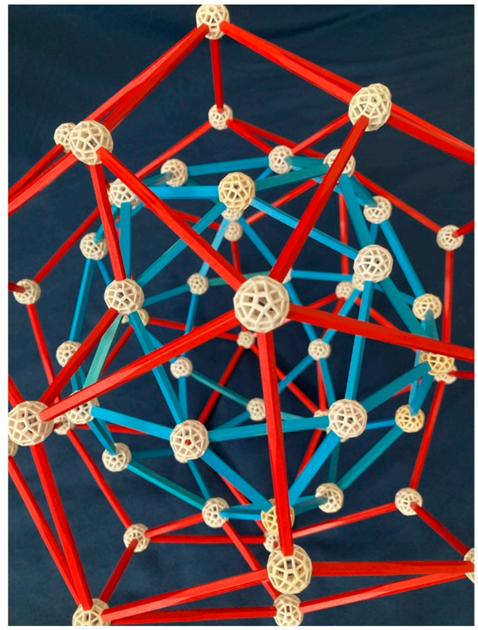
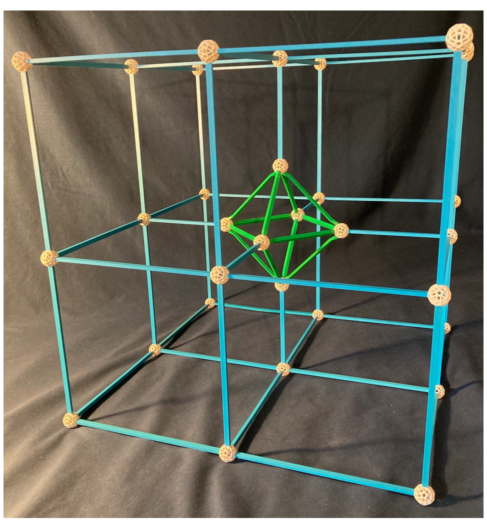
Zome Geometry
Solid Relations
"Uncle Bob"



Symmetries: There are many types of symmetries, and this is not a textbook, so we will limit ourselves to symmetries prevalent in our 3-D models of Platonic and Archimedean solids.

Rotational Symmetry: There is an old gag that goes: I can turn around so fast that you won't evenbe abletosee medoit wanna see it again? I was never that fast, but if I did turn completely around I would look pretty much the same before and after. We say that I am an object with 360-degree rotational symmetry, and that is not very interesting.

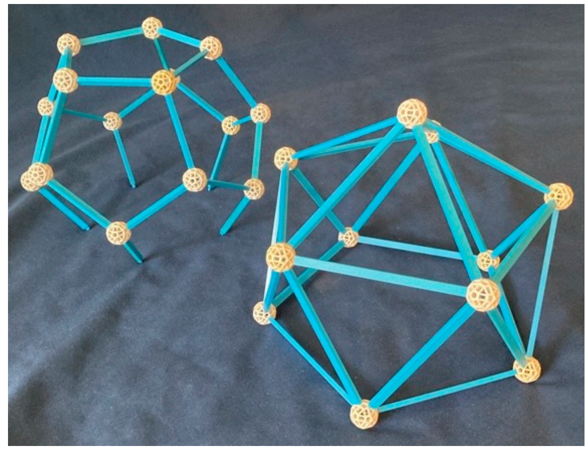
Square Symmetry: If my car had squares wheels attached to its axles, those wheels could rotate 90 degrees and look about the same to you (ignoring any brand name, that is). We say that squares have 90-degree rotational symmetry, and that the axles are 4-fold axes of symmetry — and to ponder square wheels is also pretty dull.



Cubes would have this 4-fold symmetry, of course, and being of a higher dimension, just might have some extra symmetries as well. You might want to investigate. Housed in a large cube, here is another Platonic solid [see Zome Perfect] that shares some of the cube's symmetries. It has eight faces. Can you name it?

The octahedron and the cube also possess 3- and 2-fold symmetries if you choose the axes of rotation to join opposite vertices, or the centers of opposite faces or opposite edges.

Icosahedral Symmetry: Another pair of Platonic solids share symmetries. Depending on how you hook an icosahedral wheel to the axles (axes), it exhibits 180-, 120-, and 72-degree rotational symmetries. Seventy-two degrees is one-fifth of a full turn, and you can see in these partial models how the icosahedron and the dodecahedron are intimately connected with five-sided pentagons. More on this pair in a bit.



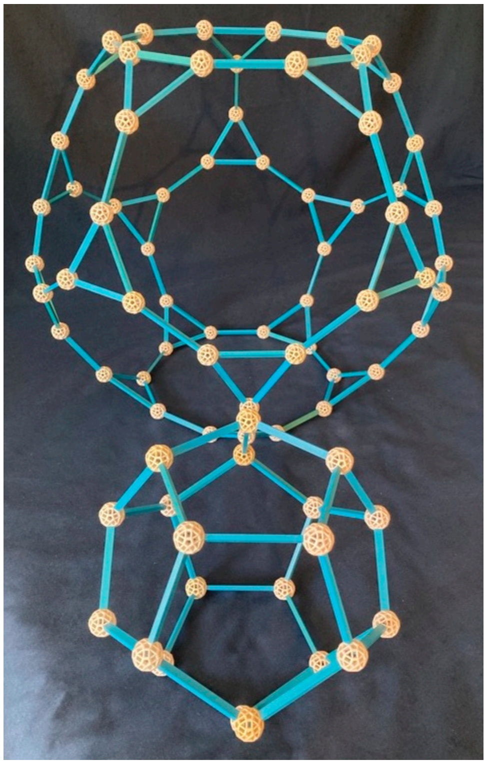
Truncation Revisited: You may recall that last month we demonstrated in rather lurid fashion that an Archimedean solid can be produced by lopping off the corners of a simpler solid. The truncated cube [3, 8, 8] was created [see Archimedes Steps Up] and proudly displayed here.

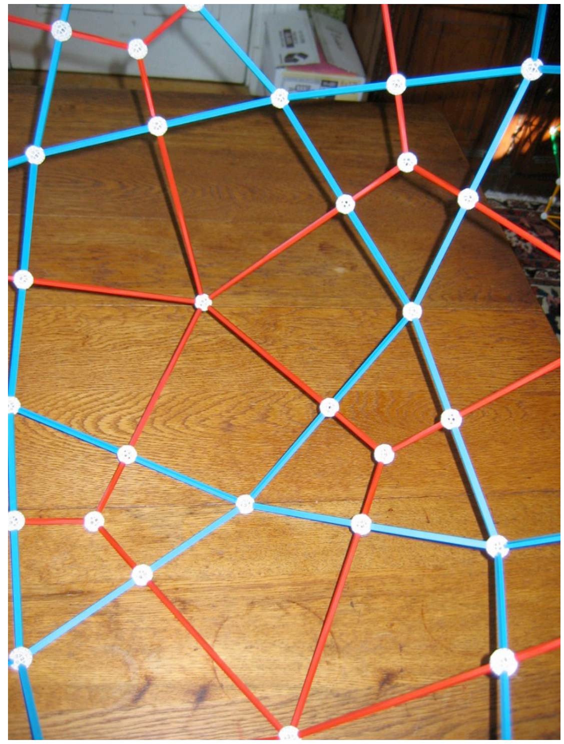
You might suspect that a truncated solid shares some symmetries with the original and you would be correct. For another example — as seen below left, even though the pair were made in different sizes, the dodecahedron, has its faces in the exact positions that the decagons have in its truncation, the Archimedean [3, 10, 10].

Duality: An even stronger bond between two solids is duality.

The compounded concoction, shown below on the right, was too large for the camera it seems, but its detail reveals much about a solid and its dual. These two duals should be familiar by now. In blue we see a portion of the Archimedean [3, 5, 3, 5], and the red rhombuses belong to the rhombic triacontahedron that we analyzed in "The Teaser."

Observe that one solid has its vertices centered over the faces of the other, and that in order to connect with an adjacent vertex the edges must be mutually perpendicular.

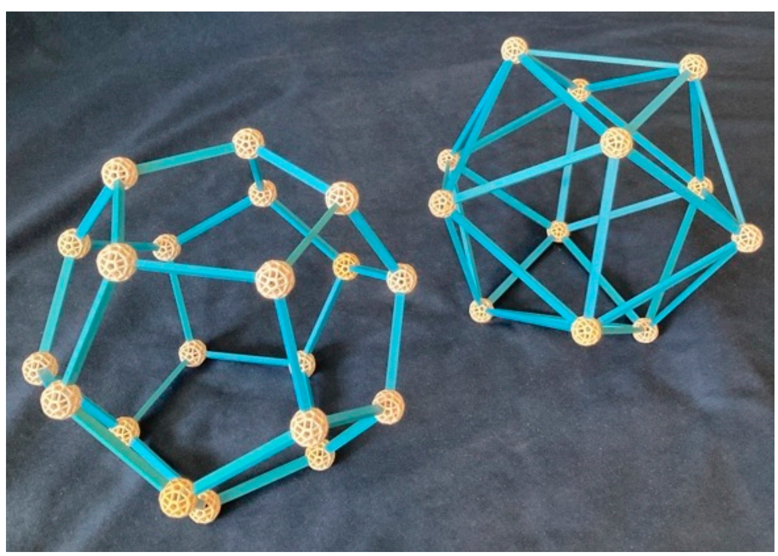




This video clip shows the two solids at different scales but with the vertices oriented over the faces in all cases.

The Archimedean solids all have duals but, as already shown, the duals are not necessarily Archimedean themselves. These duals constitute a set called Catalan solids. You can see more examples at this website.

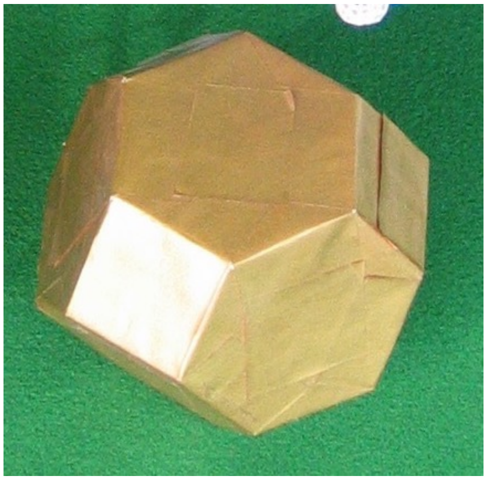
What about the duals of the Platonic solids? Those five are such an exclusive set, they keep duality "in the family" and serve as duals for one another. The picture at right shows the dual solids dodecahedron and icosahedron. Take a long look to convince yourself that the roles of faces and vertices are exchanged from one to the other.



Tasks

Solutions on page 5

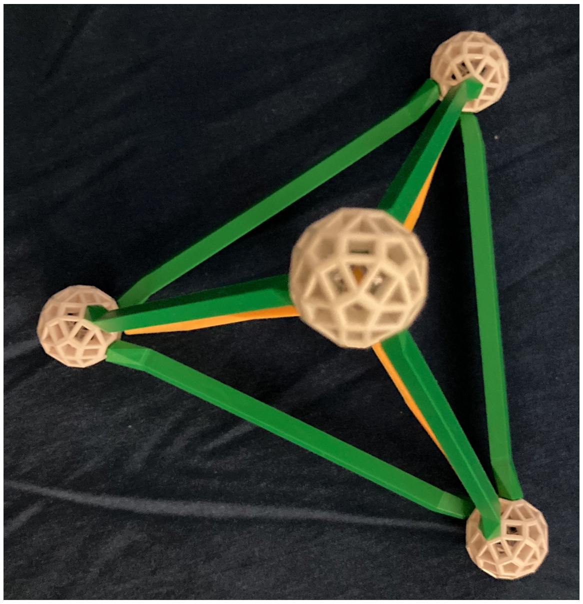
1. Dual solids must have each of their vertices centered over the faces of the other. If the icosahedron has 20 faces and 12 vertices, how many faces and vertices has the dodecahedron? How do the numbers of edges compare in these two? Refer back to the data in “Zome Perfect” and tell which solid is the dual of the cube.



2. Can you name the Archimedean solid shown at right with bracketed numbers? Refer to “Archimedes Steps Up” if needed. It is also a truncation. There are six square faces. Imagine extending the edges of the hexagons to points directly above each square. What were the faces prior to truncation? What shape was truncated to produce this one?

This solid has a surprising property. Copies of it can fit together to pack all of space just as tightly as cubes can!

3. Can you name the two solids in this video clip? What does the movie suggest about the symmetry and about the duality of the pair?



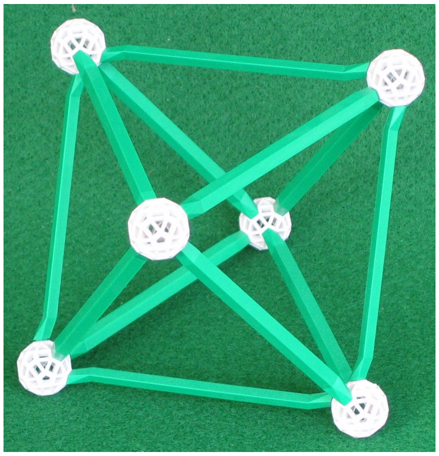
4. To this point we have paired up four of the five Platonic solids in dual relationships. What is the dual of the fifth, the tetrahedron?

Solutions

1. The counts of faces and vertices in a dual pair of solids are reversed. The dodecahedron has 12 faces and 20 vertices – the icosahedron, the opposite. The Euler number [see Zome Perfect] dictates that duals have the same number of edges, 30 in the case of this pair: 12 + 20 – 30 = 2.

The octahedron with 8 faces and 6 vertices is the dual of the cube.

2. The solid wrapped in gold paper is the Archimedean [4, 6, 6]. It can be achieved with a truncation of the octahedron, shown at the right.



3. The truncated dodecahedron, also known as Archimedean [3, 10, 10] appears to share symmetries with the icosahedron inside it; however, these two are not duals. This video shows faces of one which lack vertices of the other.

4. Staying within the Platonic family bounds, the tetrahedron serves as its own dual.