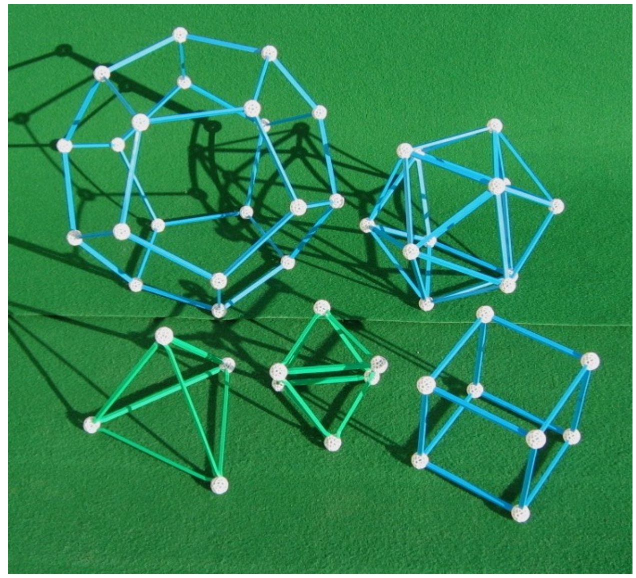
Zome Geometry
Zome Perfect
"Uncle Bob"



In the previous installment of Zome Geometry: Parts is Parts, we reinforced a principle of polyhedra — the solid figures composed of planar faces — specifically, that simple closed ones have a total angular deficit of 720 degrees over all the vertices. We also left you with the special set of them pictured above, and a table of data of the counts of their parts. The letters A through E distinguish them with solid A at the upper left and the rest succeeding in a clockwise fashion. This makes C the cube.

The condensed table follows.

	Deficit per v	Vertices	Faces	Edges	Name
A	36	20	12	30	dodecahedron
B	60	12	20	30	icosahedron
C	90	8	6	12	hexahedron
D	120	6	8	12	octahedron
E	180	4	4	6	tetrahedron

The names were withheld until now to save space — and breath. You might notice the connection between name and number of faces, as in dodeca- (twelve) and octa- (eight), leaving us to conclude that a hedron is a face.

For review purposes, let's do a check on angular deficits. The roundest of the five, the dodo (my nickname), is the one with the smallest deficit per corner, 36 degrees. Since it has 20 of those corners its total deficit is 720. The tetrahedron has the pointiest corners, where three 60-degree faces come together. Those corners differ a full 180 degrees from being flat. Total deficit is four by 180 or again 720 degrees.

These five solids we termed special. In a monumental effort to organize all of geometry logically, Euclid wrote 10 books on plane figures and properties of numbers. He finished his

Elements with three books on solids and crowned the work with a proof that the five above, the regular solids, won't be admitting any more into their club. A regular polyhedron needs to have regular (equal sided) faces which meet in exactly the same way at every corner.

One argument for "just five" goes like this: There are 360 degrees circling around every point. A polyhedron's vertex must exhibit an angular deficit and join a least three faces but no more than five. Why? Six triangles each contributing 60 degree face angles would have no deficit and lie flat. The other regular polygons have even larger face angles. Any given vertex then can accommodate three, four, or five triangles, or three squares, or three pentagons, and those specs match our set of five. Hexagons are out. Three hexagons will join at a corner, but lie flat.

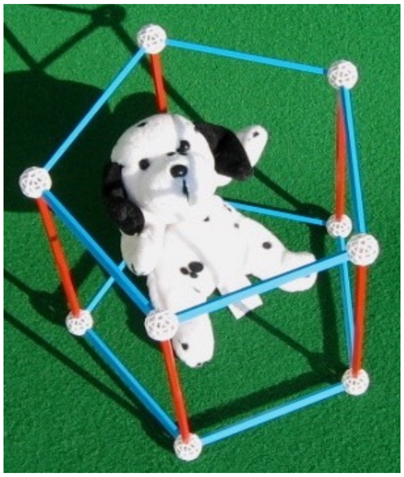
Our five are often referred to as the Platonic solids. They have achieved an ideal and a measure of perfection. We think of the square as an ideal four-sided polygon. The rectangle has equal angles but not necessarily equal sides. The rhombus has equal sides but not necessarily equal angles.

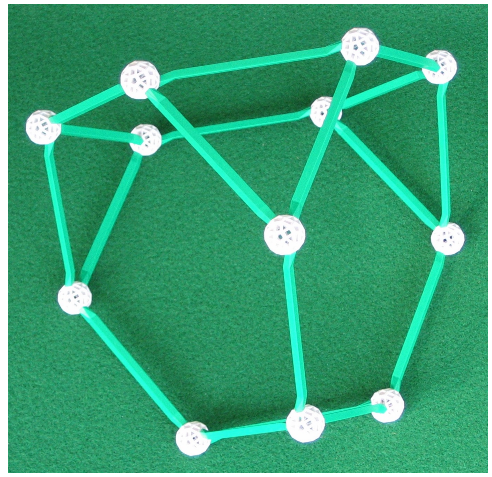
We now use our perfect set to exemplify another basic property of simple polyhedra: the Euler number (rhymes with the big ship that we wish not to have a greasy spill). Inspect the table above once more. The edges always outscore the vertices and faces, and they almost outscore both those counts put together. When we add faces and vertices, we see that edges come within two of that sum in every case. Two is the Euler number for these convex*** solids.

Angular deficit and the Euler number are tools that provide shortcuts to counting the parts of polyhedra, and we will continue on those lines in the next installment. For now, let's just test the Euler number on the following three solids.

Scroll down to view your tasks.

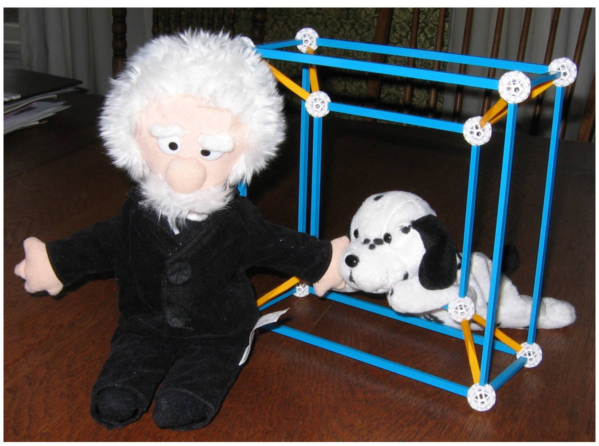
Tasks

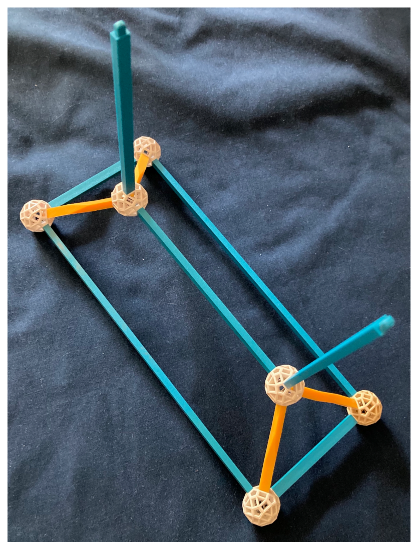




1. Sparky is confined in a closed pentagonal prison, sorry, prism. The base counts as one face. Report the counts of faces, vertices, and edges, and verify the Euler number.

2. An Archimedean solid of hexagons and triangles. Same tasks.





3. Did you read the fine print*** near the end of the article? A convex solid is well rounded — meaning no dents or dimples. Our last solid is a window frame enclosed by rectangles and trapezoids. Albert, the Person of the 20th Century, obscures somewhat, but the detail of the base (at the right) should bring clarity. Perform the same counting tasks.

Solutions on page 4.

Solutions

1. Faces, vertices, and edges are 7, 10, and 15 respectively. Euler number: $7+10-15 = 2$.

2. F, V, and E are 8, 12, and 18. Euler number: $8+12-18 = 2$.

3. Our window frame is not convex. It has a huge hole in it. Another fact to note is that the triangles are interior faces and do not count. Triangle edges do count.
F, V, and E are 12, 12, and 24. The Euler number is $12+12-24$, or zero.