Alive at 25!

Posers Celebrating 25 Years (detailed in the Blog)

Uncle Bob

Solutions begin on page 5.

1. If A is worth a penny; B, two cents; C, three; and so on. How many 25-cent common English words of length five letters can you find? Example: DECAL has a value of 4, 5, 3, 1, and 12, or 25 cents.

2. If the state sales tax is one-fifth of one-fifth of your purchase amount, what is the tax rate in percent?

3. In a magic square each row, each column, and both diagonals will sum to the "magic" total. At the right is a 3x3 magic square with 5 in the center and a magic total of 15.

8	1	6
3	5	7
4	9	2

Can you construct a 3x3 magic square with 25 in the center? What is its magic total? [Hint: Replace 4, 5, and 6 with 24, 25, and 26 and look for a pattern.]

4. Can you construct a second magic square around 25 with an entirely different supporting cast of eight numbers?

5. 25 can be expressed as 5 + 5 + 5 + 5 + 5. Can you find other ways to express 25 using exactly five digits of 5 and any mathematical operations? Example:

$$25 = (5 - 5) \times 5 + 5 \times 5$$

6. Dividing a certain 5-digit number by 25 results in a quotient missing the first digit. In the figure at the right J, K, L, M, and N can be any values zero through nine. Can you reconstruct the example?

$$\begin{array}{r} \text{KLMN} \\ 25\overline{)\text{JKLMN}} \end{array}$$

7. It's a Trap! Pick any number you wish. If you are not so hot at arithmetic, pick a small number. Add seven to the number. Multiply the sum by ten (that's easy enough) and then add 25 to the product. Next, divide by 5. [Only four more steps.] When you have the quotient, deduct 14, and then take half of the difference. This halving may result in a fraction. Not to worry; just subtract your original number and then multiply what's left by ten again. Try this again with a different starting number.

8. We're making a cake to celebrate the Puzzle Corner's 25th anniversary. It has a circular top upon which we want to place 25 candles. We'd like one or more rings of candles around a central candle. We'd also like the candles to radiate out like spokes on a bicycle wheel. For candles to be lined up like that, there needs to be an equal number of them in each ring.

If we had just one ring it would have 24 candles and be a bit crowded. Make a plan and a sketch for the candle arrangements for 2, 3, and 4 rings around the center. Make a note of the number of spokes resulting from each plan.

9. On a recent school field trip, I in my Prius was to lead a bus carrying Violet and twenty-four of her classmates to the museum. The bus became tied up in traffic and lost me at an intersection. The driver stopped and pondered. For reasons unknown, at that juncture Vy got off the bus and Elle, having missed the departure at school, boarded.

Use the letters in "twenty-five," remove Vy and add L, do some re-arranging and get back on my tail.

10. Thirty-six, or 6 x 6, is a square number with a digit sum of 3 + 6 or 9. We seek a square number that has a digit sum of 25.

11. Another Trap! From the grid at right choose any number and eliminate the rest of its row and the rest of its column from the game. Choose a second number from what remains and eliminate its row and column buddies. Likewise choose a third and then a fourth number. Find the sum of your chosen numbers.

-3	-1	3	1
4	6	10	16
1	3	7	13
3	5	9	15

12. Queen Attack. The queen in chess has unlimited movement up, down, left, right, and diagonally over the board. In a 5x5 board, try to place five queens so that they are out of one another's crosshairs. In the figure at right one queen has been placed and x's show squares under her attack.

x	x	x		
x	Q	x	x	x
x	x	x		
	x		x	
	x			x

13. Latin Square from a Queen Launch. In a 5x5 grid, place the digits 1, 2, 3, 4, and 5, so that each of them is represented in each row and column. [Suggestion: Replace the five queens from your solution to #12 above (Queen Attack) with the digit one. Look for a pattern to place the other digits.]

14. Rep-unit numbers are composed of the digit one exclusively. The number 111 is the third rep-unit and is symbolized by R3. Without using machine assistance, find a rep-unit factor of R25 other than the trivial ones — R1 and R25 itself. [Suggestion: Try divisions of smaller rep-units, and look for the principle involved.]

15. You have 25 jars. Jar#1 contains one jelly bean; Jar#2 has two jelly beans; Jar#3 has three, and so on up to Jar#25 with the mother load of 25 beans. You are to plan moves to remove all the beans, but a move has this restriction: from your choice of a set of the jars you must take an equal number of beans. For example, From jars #21 through #25, you could remove 21 beans, leaving those jars with 0, 1, 2, 3, and 4 beans. Make a plan that removes all beans from all jars in five moves. There is more than one solution.

16. Complete the sequence: 18K, 18K, 9K, 3K, 750, 150, ___.

17. In decimal (base-10) we count 1, 2, 3, 4, and 5. In binary (base-2) we count 1, 10, 11, 100, and 101. The ones digits indicate the unique set of powers of two that make up each number. Write the decimal 25 in binary form.

18. Complete the proportion. 0.5 is to 25 as 25 is to ___.

19. A train left Philadelphia at 11:45 a.m. This is NOT a joke. Traveling one time zone to the West, it made Chicago in 25 hours and 25 minutes. What time did it arrive?

20. All round questions. I have a bank with 85 quarters inside. Rather than calculate its value at first, we will estimate it using rounded numbers.

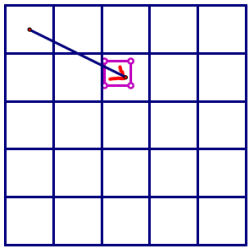
a. Round up or round down? Is the bank value closer to $27.00 (0.30x90) or to $16.00 (0.20x80)?

b. Round in or round out? Is the bank value closer to $24.00 (0.30x80) or to $18.00 (0.20x90)?

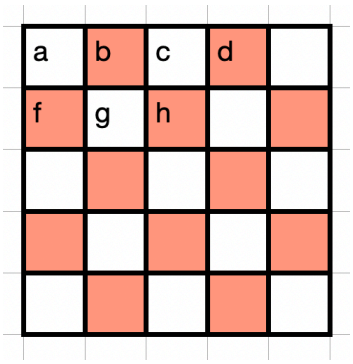
21. We hear a lot about exponential growth in the news. The sequence 1, 4, 7, 10, 13, … does not grow exponentially; it grows arithmetically by a common booster of 3. The sequence 3, 6, 12, 24, 48, … grows exponentially by the common factor of 2. Find the difference between the 25th terms in these two sequences.

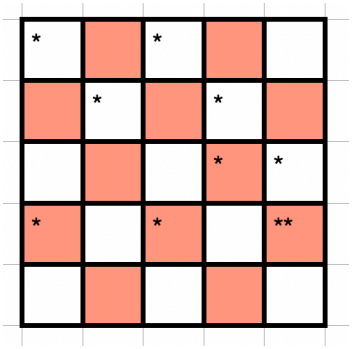
22. In regard to the previous problem, a more realistic growth factor in some circumstances would be 5% or 1.05. What is the 25th term in the sequence which begins with 3 and grows by 5%?

23. One-way quest. A knight is to visit every square once and only once in a tour of a 5x5 chess board. A knight's move traces an L, either two squares ahead, and then left or right one square; or one square ahead, and then left or right two. A suggested starting move is shown. Can you complete the tour? Hint: this tour I have in mind ends at the center square.



24. Wanna Fight? The 5x5 board below shows an obvious way to encode the alphabet. What's that? Don't hassle me, buddy! I know there are 26 letters in the English alphabet. Well, pfui! The letters are, after all, of varying importance. I've encoded a ten-letter word in the second grid. This grid skips over the letter "J," or uh … maybe it skips Q, or is it X or Z? I can't remember which one. Wanna make something of it? Then tell me what the word is, pal.





25. The three-digit number 376 has an interesting property. Any whole power of 376 will end in the digits 376. For example 376 cubed is 53,157,376. This number seems stuck on itself, and yes there is one other three-digit example. Can you find it. [Suggestion: To narrow your search, find which single-digit and which two-digit numbers have powers with this property.]

_____. _____. _____. _____. _____. _____.

Solutions

1. LACED, EDGED, DICED, and GAFFE are worth 25 cents. There may be others.

2. 4%. 1/5 of 1/5 = 1/25 or 4/100.

3.

28	21	26
23	25	27
24	29	22

4. Other answers exist.

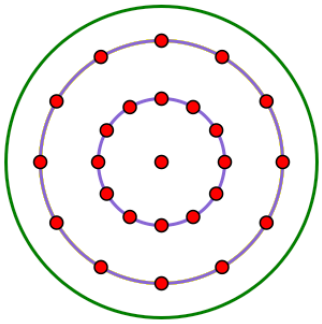
40	5	30
15	25	35
20	45	10

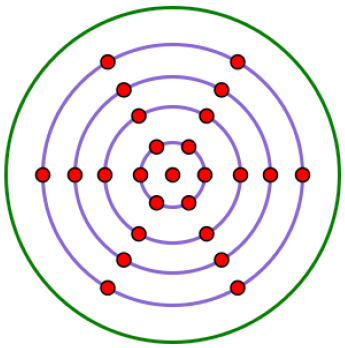
5. 55 - (5 x 5) - 5
5^(5/5 + 5/5) which is 5-squared
5 - 5 + (5! + 5)/5 [NB: 5! is five factorial, or 5 x 4 x 3 x 2 x 1
5 + (5/.5) + (5/.5)
5 + (5 + 5 + 5)/.5
5 x (5/.5) - (5 x 5)
There are likely others.

6. 62500/25 = 2500.

7. The result should always be 25. For example, beginning with 2, and following instructions, the sequence of intermediate results is 2, 9, 90, 115, 23, 9, 4.5, 2.5, 25.

8. Cake and candles. Shown are sketches of 2-ring and 4-ring arrangements.





9. The letters anagram to "I went left."

10. The square of 67 is 4489. The digits sum to 25. The square of 76 also satisfies.

11. Your choices under these conditions will sum to 25 always.

12.

Q				
			Q	
	Q			
				Q
		Q		

13.

1	2	3	4	5
3	4	5	1	2
5	1	2	3	4
2	3	4	5	1
4	5	1	2	3

14. R5 is a factor of R25. In longhand:

```
                 1 00001 00001 00001 00001
11111 ) 11111 11111 11111 11111 11111
```

If m divides n, then Rm divides Rn. Example, R2 divides R8:

```
        01010101
11 ) 11111111
```

15. One solution is to remove 13 beans from jars #13 to 25, leaving at most 12 beans in any jar. The second move would take 7 beans from the jars with at least that number, leaving at most 6 beans in any jar. The basic idea is to operate on half of the jars at each move. Continuing the above, moves 3, 4, and 5 would be to take respectively 4, 2 and a single bean from jars that contained those amounts.

A second plan might be to take 16 beans from jars #16 to 25, and can you take it from there?

16. 25 (one-sixth of 150)

17. 11001. Instead of powers of ten, binary places are powers of two. Our answer can be read 16 + 8 + 0 + 0 + 1 in decimal portions.

18. 1250. 25 is 50 times greater than 0.5. 50 x 25 = 1250

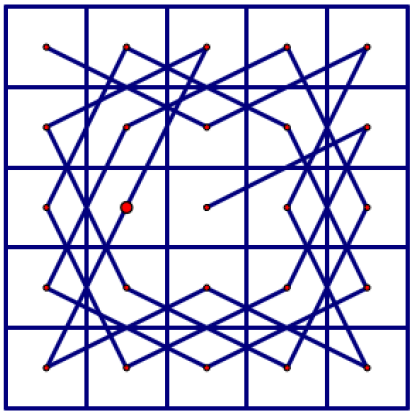
19. Ten minutes after noon (Central Time) the next day.

20. Simple calculation reveals that rounding both 25 and 85 down results in a better approximation than rounding both up which seems to magnify the error. The question of rounding in or out assumes that there is a difference in the tens and higher places. Rounding in gives a better result. A little algebra can show this is always the case.

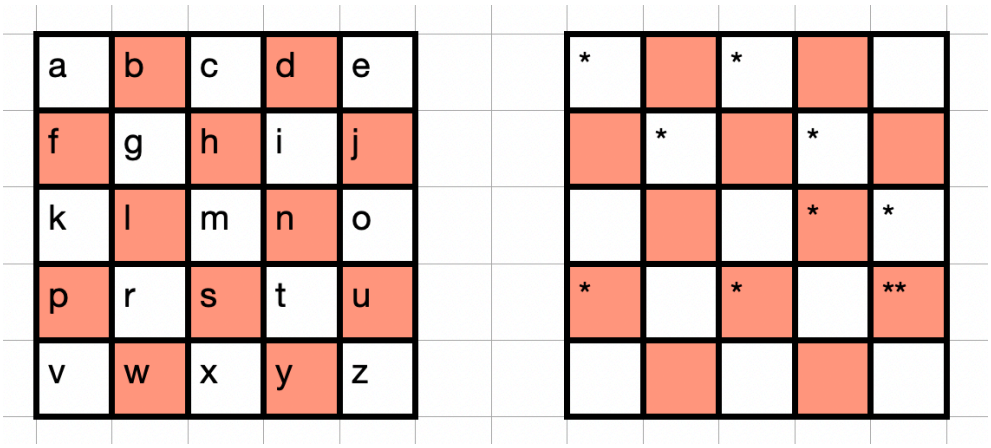
21. The 25th term in the first sequence beginning with 1 is 24 x 3 + 1 = 73. The 25th term in the sequence which doubles each time beginning with 3 is 50,331,648. The difference is vast.

22. 3 x 1.05^24 = 9.6753 approximately. Now imagine a problem in our world that is at level 3 now and is allowed to grow 5% per annum. It will be at a level near 10 in 25 more years.

23.



24. PUGNACIOUS. The grid skips Q, and the letters A, C, G, I, N. O, P, S, U, and U can be arranged for the solution. Don't like it? Tough beans.



25. Any power of 625 will end with the digits 625.

We hope you enjoyed these. Please email Uncle Bob with tales of your follow-ups and foibles.